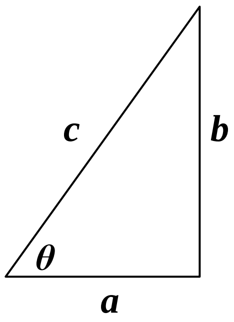
# THE TRIGOMETER.

The basic definitions for sin, cos and tan need to be extended for angles greater than $90^0$

## BASIC DEFINITIONS:



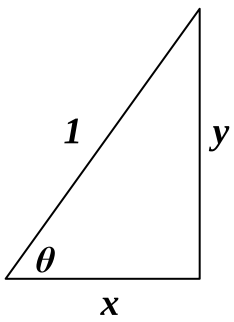

Using the old SOH CAH TOA idea :

$$\sin\theta = \frac{b}{c}$$

$$\cos\theta = \frac{a}{c}$$

$$\tan\theta = \frac{b}{a}$$

The first step is to let the hypotenuse be of length 1 unit:




Using the old SOH CAH TOA idea :

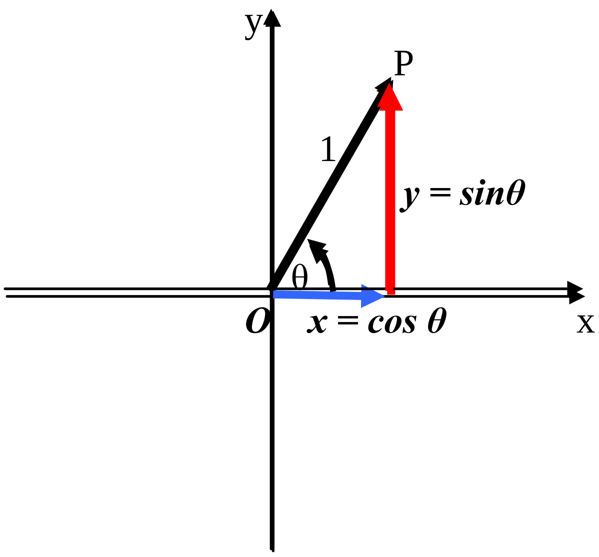
$$\sin\theta = \frac{y}{1} = y$$

$$\cos\theta = \frac{x}{1} = x$$

$\tan\theta = \frac{y}{x}$ ***this is just the same as above.***

## NEW DEFINITIONS.

**Now we imagine that OP is a unit vector which can rotate about the origin.**

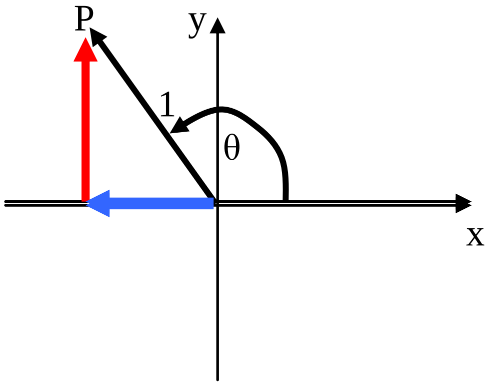



**Now sin θ is defined to be the y coordinate of unit vector OP**
**Similarly, cos θ is the x coord of OP.**

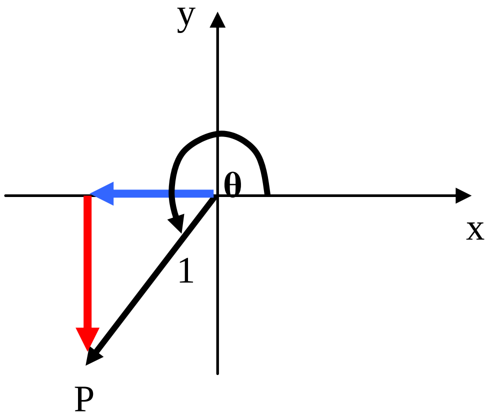
Also, since $\tan\theta = \frac{y}{x}$

then $\tan\theta = \frac{\sin\theta}{\cos\theta}$

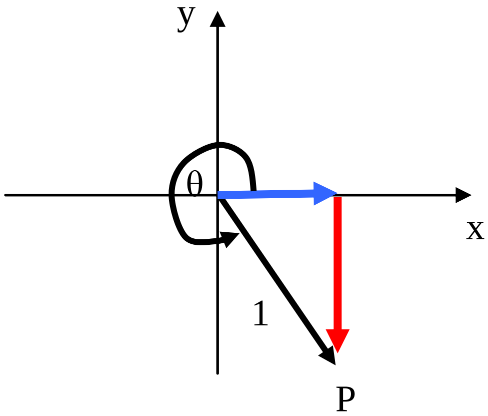
**As OP rotates around O we can produce all we need to know about sin, cos and tan of any sized angle. <u>We are now not restricted to angles less than $90^0$.</u>**



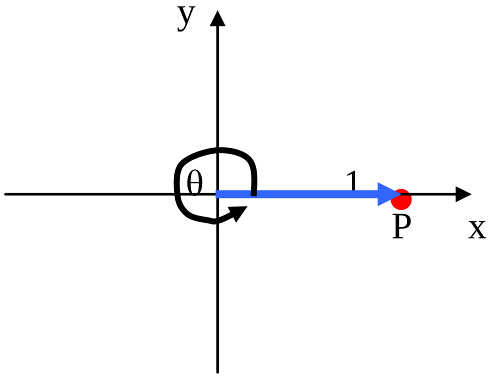

In this case suppose $\theta = 120^0$
then **sin 120 = a positive *y* value**
but **cos 120 = a negative *x* value**



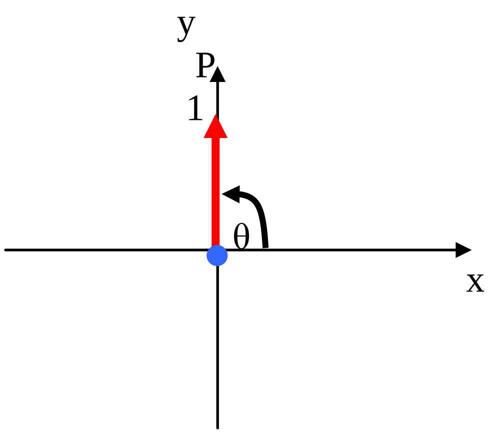

In this case suppose $\theta = 240^0$
then **sin 240 = a negative *y* value**
and **cos 240 = a negative *x* value**




In this case suppose $\theta = 300^0$
then **sin 300 = a negative *y* value**
but **cos 300 = a positive *x* value**



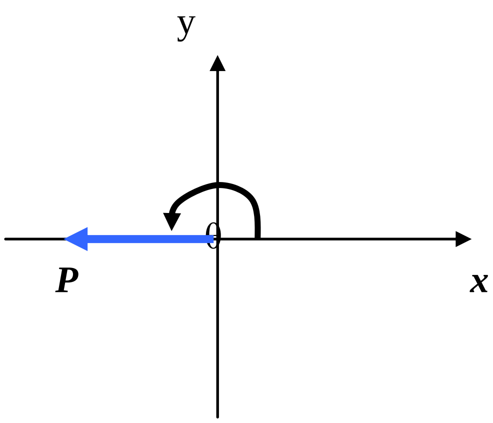

In this case θ could be $0^0$ or $360^0$
Clearly **sin θ has reduced to zero**
and **cos θ has become length 1**
because it equals OP
ie **sin 0 = 0**
but **cos 0 = 1**



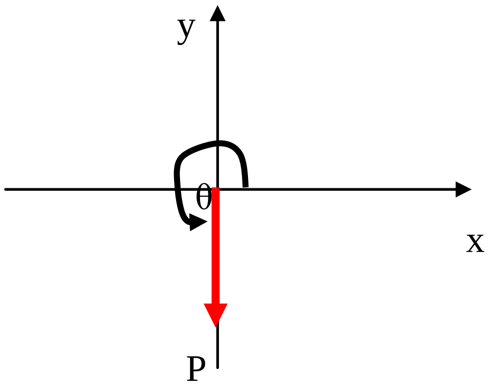

In this case $\theta = 90^0$
Clearly **sin 90 = 1**
but **cos 90 = 0**

$$\tan 90^0 = \frac{1}{0}$$

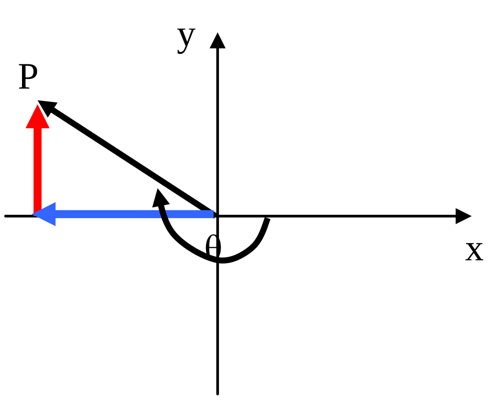
which is "undefined" or "not finite" or "non-finite" or "infinite".
Use of the term "infinity" implies a number immense size but it is not a number.
**<u>In</u>**finite means it is not a **finite** number,
(ie not a **<u>de-</u>**finite number)




In this case $\theta = 180^0$
Clearly **sin 180 = 0**
but **cos 180 = – 1** ***(remember cos θ is the <u>x coord</u> of OP = – 1 not the <u>length</u> of OP.)***




In this case $\theta = 270^0$
Clearly **sin 270 = – 1**
but **cos 270 = 0**

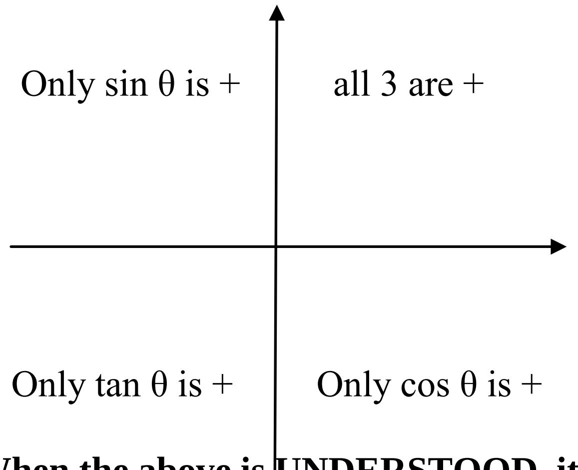



Anticlockwise angles are called POSITIVE
but Clockwise angles are NEGATIVE.
In this case suppose $\theta = -240^0$
So **sin (– $240^0$) = a POSITIVE *y* value**
but **cos (– $240^0$) = a NEGATIVE *x* value**

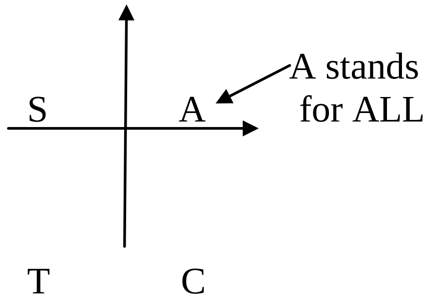
In all of the above cases $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and so $\tan\theta$ takes its sign either positive or negative from the individual signs of $\sin\theta$ and $\cos\theta$.

eg since sin 120 is positive but cos 120 is negative, then $\tan\theta = \dfrac{\text{pos}}{\text{neg}}$ = negative

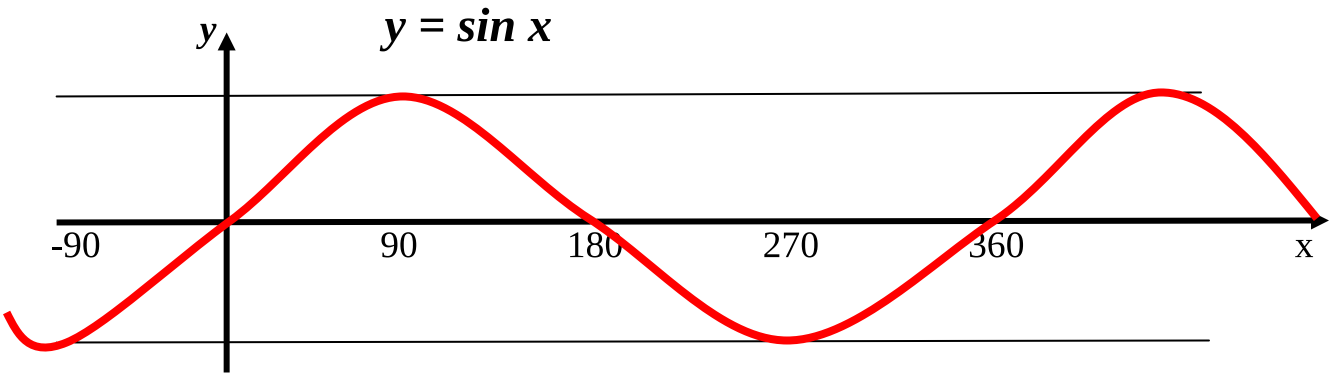
An old but good method to summarize this information on one diagram is to indicate which of sin, cos or tan is positive in the various quadrants.

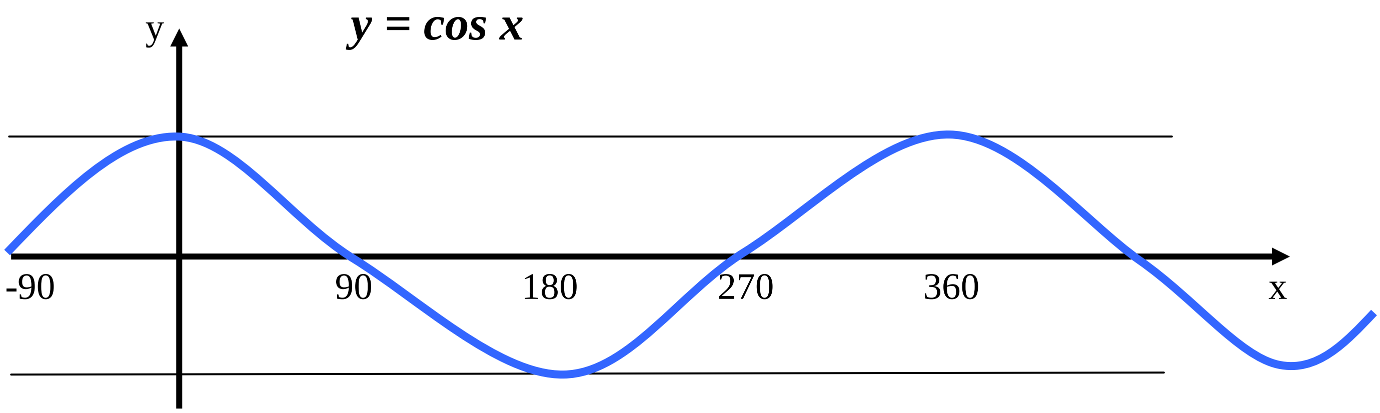



Often reduced to :

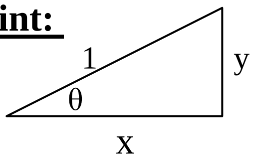



**When the above is UNDERSTOOD, it makes sense where the sin and cos graphs are <u>positive</u> and where they are <u>negative</u>.**




**<u>One last point:</u>**




Obviously $y = \sin\theta$ and $x = \cos\theta$

Also $y^2 + x^2 = 1^2$ (by Pythagoras' Theorem)

so $\sin^2\theta + \cos^2\theta = 1$